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Project Director(s) R.G. Jeroslow GTRC / ~~GIT~~

Sponsor National Science Foundation

Title "Mathematical Sciences: Integer and Semi-Infinite Programming"

Effective Completion Date: 12/31/86 (Performance) 3/31/87 (Reports)

Grant/Contract Closeout Actions Remaining:

- [X] None
- [ ] Final Invoice or Final Fiscal Report
- [ ] Closing Documents
- [ ] Final Report of Inventions
- [ ] Govt. Property Inventory & Related Certificate
- [ ] Classified Material Certificate
- [ ] Other

Continues Project No. Continued by Project No.

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Progress Report

November 1982-October 1983

In our paper [R1], we took an extremely simple game-theoretic setting - a problem which many specialists would view as "trivial" from a theoretical perspective - and showed that it exhibits a complexity which goes beyond NP, and in fact up (at least) all levels of the polynomial hierarchy. The model we studied has occured in the setting of governmental policy determination and bears directly on models for delegation of authority and the principal agent problem. To our knowledge, this is the first use of the polynomial hierarchy in an applied setting.

The purpose for our investigations in [R1] was to try to understand why it has proven to be so difficult to develop decision support for even simple contexts involving gaming. These matters are discussed in the introduction and final section of [R1]. We are now proceeding on algorithmic studies of an important special case of these models which is only NP-complete. This work requires the efforts of a part-time computer assistant.

In our joint paper [R2] with our doctoral student James Lowe we implemented an extensive computer testing of our new techniques for modelling practical problems as mixed-integer programs. In this relatively new kind of research, we do not (as yet) change the algorithm, but only the way the real-world problem is represented mathematically. The theory behind the tests of [R2] is given in our earlier paper [G1].

In two scenario-based implementations, the new model formulation techniques have proven distinctly superior to the earlier ones, both in terms of CPU times and nodes of the branch-and-bound tree. In our tests, the advantage increases with problem size. Our random problems run so far are small by industrial standards. These tests were done with a slow code, and the arrival of APEX 4 on our campus about two months ago is allowing our experiments to proceed to larger sizes and more scenario types. We also wish to use actual data. We have contacted another researcher in a consulting setting and have initiated some joint work. Ideally, we would like another one or two such contacts to help guide the development of our work.

Simultaneously, we are now extending the representation theory of [G1] in a paper [P1]. In [P1], we extend it to nonconvex nonlinear programming, explore several new mathematical issues which have arisen in furthur conforming the representations to existing production codes, and relate it to certain representations to the Shapley-Folkman Theorem.

We have been doing a fair amount of reading in artificial intelligence, out of a belief that aspects of the "expert systems" provide another application for the representation theory of [G1] and, more generally, the disjunctive methods of [G2], [G3], [G4]. This appears to be the case, though we will not be able to do computer testing in the near future. (Most of Captain Lowe's remaining time in our doctoral progam, to March 1984, will be spent in writing up earlier and current experiments for his thesis.)

However, after [P1] and [P2] we will do a paper to show several of the modelling possibilities in artificial intelligence.

Incidentally our Ph.D. dissertation was in mathematical logic and some years ago we did a paper on mechanical learning processes [G5].

We recently performed a series of small experiments on the handling of propositional logic via standard integer programming formulations with a production code (APEX). It does indeed seem that this "off-the-shelf" integer programming approach easily handles the great majority of systems with up to 300 production rules in up to 300 propositional letters, in several CPU seconds. Two independent researchers in artificial intelligence tell us that the usual AI algorithms would have difficulties with propositional logic problems of this (medium) size. After seeing this, we systematically created hard propositional problems that won't solve by the standard formulation in several minutes. Now we are proceeding, jointly with Egon Balas of Carnegie-Mellon University, to develop improved formulations for these harder problems and for much larger ones. The propositional logic issues, however, are only one aspect of applications of the new representation techniques in expert systems.

In the note [R3], we are continuing our earlier studies on parametric integer and mixed-integer programming [G6], [G7], but this time from an algorithmic perspective. In [P2] we will finalize and extend work of [R3] and submit it to publication. Our work in [R3] was motivated by an early version of [G8].

Recent Research Reports

[R1] "The Polynomial Heirarchy and a Simple Model for Competitive Analysis," March 1983 (submitted for publication).

[R2] "Experimental Results on the New Techniques for Integer Programming Formulations," with J. K. Lowe, July 1983 (submitted for publication).

[R3] "Sensitivity Analysis in Mixed IP via Subadditive Families," a preliminary report, June 1983.

Papers Currently Being Written

[P1] "Representations of Practical Problems as Mixed-Integer Programs."

[P2] "Sensitivity Analysis in Mixed Integer Programming."

General References

[G1] R. G. Jeroslow and J. K. Lowe, "Modelling with Integer Variables."

[G2] E. Balas, "Disjunctive Programming: Cutting-planes from Logical Conditions," in Non Linear Programming 2, O. L. Mangasarian, R. R. Meyer, and S. M. Robinson (editors), Academic Press, 1975, pp. 279-312.

[G3] E. Balas, "Disjunctive Programming and a Hierarchy of Relaxations for Discrete Optimization Problems," MSRR no. 492, GSIA, Carnegie-Mellon University, June 1983.

[G4] R. J. Jeroslow, "Cutting-plane Theory: Disjunctive Methods," Annals of Discrete Mathematics 1 (1977), pp. 293-330.

[G5] R. G. Jeroslow, "Experimental Logics and Δ_2^0-Theories," Journal of Philosophical Logic 4 (1975), pp. 253-267.

[G6] C. E. Blair and R. G. Jeroslow, "The Value Function of an Integer Program," Mathematical Programming 23 (1982), pp. 237-273.

[G7] C. E. Blair and R. G. Jeroslow, "Constructive Characterizations of the Value Function of a Mixed-Integer Program."

[G8] L. Schrage and L. Wolsey, "Sensitivity Analysis for Branch-and-Bound IP," August 1983.

Other Support

The Principal Investigator has no other grant support and no other grant proposals are currently submitted or in progress.

Due primarily to the desirability of finding support for doctoral students, who are essential to much of the research in this grant, we are currently developing interconnections between some of our research and issues in artificial intelligence. These efforts may come to fruition over the next six months and may result in a proposal, possibly joint with other colleagues, to an appropriate agency. If this is done, we are likely to also seek release time in order to concentrate on this research, but we are not seeking any increase in total personal compensation beyond that provided by this grant.

Publication Activity,

November 1982-October 1983

Published

"Duality in Semi-Infinite Programming," with R. J. Duffin and L. A. Karlovitz, Semi-Infinite Programming and Optimization, edited by A. V. Fiacco and K. O. Kortanek, Springer-Verlag, 1983.

Accepted for Publication

1. "Uniform Duality in Semi-Infinite Convex Optimization," to appear in Mathematical Programming.

2. "Cluster Sets of Vector Series," with D. F. Karney, to appear in Advances in Applied Mathematics.

3. "Extensions of a Theorem of Balas," with C. E. Blair, to appear in Discrete Applied Mathematics.

Submitted for Publication[1]

1. "The Polynomial Heirarchy and a Simple Model for Competitive Analysis."

2. "Experimental Results on the New Techniques for Integer Programming Formulation," with J. K. Lowe.

Papers in Preparation

1. "Representations of Practical Problems as Mixed-Integer Programs."

2. "Sensitivity Analysis in Mixed Integer Programming."

[1]Other items listed in the vita under this heading were done prior to November 1982.

Funds Remaining in This Grant and the Previous Grant at the End of the Current Grant Period

We estimate between 0-5% of funds remaining in the current grant increment (i.e. for the first year) at the end of the grant period. The exact amount will depend upon our finding and/or continuing suitable graduate students as part-time programmers. All other expenses will be as anticipated.

We estimate that essentially no funds (0-1%) will remain in our previous grant ECS-8001763 by December 31, 1983.

Progress Report

November 1983-October 1984

In the papers [R1], [R2], [R3] done this year in connection with the grant, we have: (1) Characterized an extension of linear mixed-integer representability, which we call "bounded convex representability"; (2) Shown (by examples and exact results) applications of bounded convex representability, the limitations of conventional modelling for either/or constraints, and the value of parametrized forms of disjunctive representations; (3) Shown the differences in representing functions that appear in constraints, rather than simply objective functions of mixed-integer programs, and exactly characterized both classes of functions; (4) Developed constructions of representations for the union, intersection, sum, cartesian product, and projection of sets, and defined the 'composite constructions' which derive from repeated use of these basic constructions; (5) Exactly characterized the linear or convex relaxation obtained from the composite constructions; and given a broad sufficient condition for these relaxations to be "best possible" (i.e. to be the convex span of the set represented), and to retain that optimality property in lower nodes of a branch-and-bound search tree.

In an appendix to our earlier joint paper [G2], we showed how our modelling techniques accounted for the tightness of the linear relaxation for the 'disaggregated formulation' that was emphazized in [G6]. In addition, these modelling techniques led to two new formulations which proved experimentally to be superior in [G2].

These papers [R1], [R2], [R3] are a continuation of earlier joint work [G1], [G2] and are the beginning of a projected series of papers to be continued in [W1], [P1], and [P2]. We are in the process of exploring superior ways of representing practical problems as mixed-integer programs, by defining structural features and using these in novel ways. The techniques derive from or extend earlier work on the disjunctive methods [G3], [G4], [G5].

In [W1] we will be reporting on uses of these modelling techniques in propositional and some predicate logic contexts, in some instances also with "confidence factors." The research in [W1] is intended to show applications of mixed-integer programming as inference engines in significant applicants contexts of artificial intelligence. The full utilization of these techniques will require further information in [P1] and [P2].

FINAL REPORT

INTEGER AND SEMI-INFINITE PROGRAMMING

R.G. Jeroslow

Report for the Period July 1, 1983 through December 31, 1986

Prepared for

National Science Foundation

Under

NSF Grant MCS-8304075

GEORGIA INSTITUTE OF TECHNOLOGY

A Unit of the University System of Georgia

Atlanta, Georgia 30332

GTRI

GEORGIA TECH RESEARCH INSTITUTE

FINAL REPORT

to the

National Science Foundation

for

NSF Grant MCS-8304075

"Integer and Semi-infinite Programming"

TABLE OF CONTENTS

Attachments

NATIONAL SCIENCE FOUNDATION Washington, D.C. 20550	FINAL PROJECT REPORT NSF FORM 98A	
PLEASE READ INSTRUCTIONS ON REVERSE BEFORE COMPLETING		
PART I—PROJECT IDENTIFICATION INFORMATION		
1. Institution and Address Georgia Tech Research Institute Atlanta, GA 30332	2. NSF Program Math Sciences	3. NSF Award Number MCS-8304075
	4. Award Period From 7/1/83 To 12/31/86	5. Cumulative Award Amount $101,322
6. Project Title Integer and Semi-Infinite Programming		

PART II—SUMMARY OF COMPLETED PROJECT *(FOR PUBLIC USE)*

We have extended the theory of mixed-integer programming representations of practical problems, such as arise in lot-sizing, shop loading, distribution of effort, and commodity distribution problems. Building on the approach to representability due to R. R. Meyer, by use of disjunctive methods we have defined a new concept of problem "structure". This concept is in conformity with the manner in which problems actually arise, from choices of alternatives (union), selection of subactivities (projection), juxtaposition of constraints (intersection), decentralization (Cartesian product), addition, and other algebraic operations. Distributive laws for unions and intersections are used to obtain alternate formulations of the same constraints, with increasingly tighter linear relaxations, in a manner forming a lattice structure.

We have also provided interconnections between mixed-integer programming and logic-based approaches to decision support, and continued our development of logic algorithms. These latter developments are potentially useful as inference engines for expert systems which go beyond the Horn clause framework.

PART III—TECHNICAL INFORMATION *(FOR PROGRAM MANAGEMENT USES)*

1. ITEM *(Check appropriate blocks)*	NONE	ATTACHED	PREVIOUSLY FURNISHED	TO BE FURNISHED SEPARATELY TO PROGRAM Check (✓)	Approx. Date
a. Abstracts of Theses		X			
b. Publication Citations		X			
c. Data on Scientific Collaborators	X				
d. Information on Inventions	X				
e. Technical Description of Project and Results		X			
f. Other *(specify)*					

2. Principal Investigator/Project Director Name *(Typed)*	3. Principal Investigator/Project Director Signature	4. Date
Robert G. Jeroslow	[illegible]	[illegible]

Technical Description of Project and Results

We have extended the theory of mixed-integer programming representation of practical problems (begun by R. R. Meyer) and introduced a new concept of problem "structure". We also have provided interconnections between mixed-integer programming and logic-based approaches to decision support, and continued our development of logic algorithms.

In more detail:

(1) Representability for certain nonlinear constraints has been characterized;

(2) Representability of the graph of piecewise-linear functions on polyhedral domains has been characterized;

(3) A concept of structure has been introduced, which follows the algebraic generation of constraint sets, and in this context important results on distributive laws have been obtained;

(4) Results on a certain kind of simplification in some representations, involving significant size reductions, have been obtained;

(5) We have provided compact and sharp (i.e., best linear relaxation) representtions for certain specialized constraints arising Production and Operations Management;

(6) The concept of representability has been generalized and axlomatized via the concept of an embedding, leading to a generalization of Benders' partitioning;

(7) Tie-ins between the standard integer programming representation of propositional logic and a well-known logic algorithm have been shown, together with several related results;

(8) Reductions of predicate to propositional logic have been given, involving increasingly accurate propositional "approximations" to predicate logic, in a manner which makes mixed-integer programming and propositional logic techniques usable for query answering;

(9) An algorithm for propositional logic has been efficiently coded, which is far faster than earlier algorithms;

(10) An approach has been developed for achieving certain inductive defintions by mixed-integer programming; it is applicable to certain implementations of imprecise reasoning.

In the Technical Description below, a section is devoted to each item cited (1)-(10).

Our lecture notes for the talks given at Rutgers University last May [61], as part of the Advanced Research Institute on Discrete Applied Mathematics (ARIDAM), go in further detail for most of these ten points. Following each section title, we cite the relevant lecture. The notes are an attachment for this report.

1. Representability for Nonlinear Constraints, and Some Extensions of Work for the Previous NSF Grant ECS-8001763 (Lecture 4)

A set $S \subseteq R^n$ is bounded convex representable (b.c.r.) if there is a vector function $f(x;y)$ of variables $x \in R^n$ and "auxiliary variables" y, which is co-ordinate-wise homogeneous, closed and convex, with no values of $-\infty$, meeting a regularity condition (see below), and a vector b and index set K for a subset of indices of y, with:

$$x \in S \leftrightarrow \text{there exists } y \text{ with } y_k \in \{0,1\} \text{ for } k \in K \text{ and } f(x;y) \leq b \tag{1.1}$$

The regularity condition on f is as follows:

$$f(0;y) \leq 0 \rightarrow y = 0 \tag{1.2}$$

The characterization result for b.c.r. sets is:

Theorem 1.1: [54]

A set S is b.c.r. iff $S = S_1 \cup \ldots \cup S_t$ is a finite union of closed, convex sets S_i with the same recession directions (i.e., $\mathrm{rec}(S_i)$, $1 \leq i \leq t$, is independent of i).

By an example in [54, page 15] we show that the regularity condition is needed to obtain the above characterization.

This result continues our earlier work on bounded mixed-integer programming representability (b-MIP.r), in which we specified that f(x;y) be a linear transformation, and we did not need a regularity condition (1.2).

The b-MIP.r concept subsumes all the known instances of mixed-integer representability of various kinds of constraints (e.g., fixed charges, piecewise linear functions, either/or constraints, etc.). It is an outgrowth of work by R. R. Meyer [77], [78], [79], [80]. The b.c.r. concept has also accounted for fixed charges added to convex cost functions, either/or convex constraints, and other common uses of bounded integer variables in which each subproblem (with fixed values of the binary variables) is a convex program.

The "if" part of Theorem 1.1 is established by a direct construction of a representation $\underline{S}$ for S, which is based on the disjunctive methods [1], [2], [3], [4], [49], [50]. In our later paper [55] we show that the relaxation of the representation $\underline{S}$ we construct is sharp, i.e., we have

$$\mathrm{Rel}(\underline{S}) = \mathrm{conv}(S) \tag{1.3}$$

where

$$\mathrm{Rel}(\underline{S}) = \{x \mid \text{for some } y \text{ with } 0 \leq y_k \leq 1 \text{ for } k \in K,\ f(x;y) \leq b\} \tag{1.4}$$

In particular, clconv(S) = conv(S) for S b.c.r. (here conv(S) respectively clconv(S) is the convex span resp. the closure of the convex span of the set S). For convexity concepts see [93], [100]. Since Rel($\underline{S}$) $\supseteq$ conv(S) for any representation $\underline{S}$ of S, sharpness (1.3) is an optimality condition. In view of the fact that most algorithms for solving a program proceed by use of a series of relaxations of the program, it is desirable that the relaxations be as accurate as possible, i.e., that they be sharp. While some commonly-used representations (e.g., simple fixed charges) are sharp, others are not (as we showed in [63] for our earlier grant). There can be substantial computational advantage to the use of sharp representations, even when they are substantially larger than nonsharp ones (as we showed earlier [63]).

Of course, while sharpness of a representation is very desirable, there are trade-offs with the size of the representation. We will discuss this issue in Section 3 below. Let us define the starred recession cone of a representable set by:

$$\mathrm{rec}^*(S) = \{w \mid \text{for some } x \in S,\ x + \lambda w \in S \text{ for all } \lambda \geq 0\} \tag{1.5}$$

$$(= \{w \mid \text{for all } x \in S \text{ and } \lambda \in 0,\ x + \lambda w \in S\})$$

Then rec*(S) = rec(S) for $S \neq \emptyset$ closed and convex [55] and in fact

$$\text{rec}^*(S) = \{w \mid \text{for some } y \text{ with } y_k = 0 \text{ for } k \in K, \; f(w;y) \leq 0\} \tag{1.6}$$

Using (1.6) greatly facilitates the proofs of standard results on sufficient conditions for algebraic operations on closed, convex sets to yield closed, convex sets (see [55]). In fact, the latter results are generalized to b.c.r. sets. Essentially, the compactness arguments typical of such results have been compressed into the proof of Theorem 1.1, and what remains are only algebraic calculations using (1.2) and (1.6); see [55] for details.

In addition to the characterization result of Theorem 1.1, the following work was concluded, as a direct outgrowth of research continuing from the previous grant ECS-8001763:

(1) We confirmed the value of the proximity of the linear relaxation to the set modeled, as a measure of solution difficulty by MIP for branch-and-bound codes. This was done by taking a series of fixed charge problems and pairing each with a fixed benefit problem, in which the charge was simply reversed in sign to become a benefit. While both function types have sharp representations, the convex span of fixed charge problems is not close to the set modeled, while a sharp representation of a fixed benefit problem is very accurate. As we expected, the fixed benefit problems were very easily solved (often in the linear relaxation), while the fixed charge problems were hard to solve. For details on our experiment, see Chapter Five of [71]. (For a definition of fixed benefit functions, see Lecture 1 or see Section 3 of [54].)

(2) In [54], we introduced a general "parametrized" construction of representations, which subsumes both of the earlier "polyhedral" and "extreme point" representations of [62]. We also gave an example in which a compact, sharp parametrized representation existed, even though the sizes of both the "polyhedral" and "extreme point" representation are exponential (see the last example in Section 3 of [54]).

2. Representability of the Graph of Functions (Lecture 1)

When a function $f(x)$ occurs in a mixed-integer-program (MIP) only additively in a positive manner as "+ $f(x)$" in the (minimizing) objective function, a representation of its epigraph $\mathrm{epi}(f) = \{(z,x) | z \geq f(x)\}$ is generally needed. The same kind of representation also suffices if, in addition, "+ $f(x)$" occurs in the constraints of the less-than-or-equal-to ($\leq$) type. However, if the function occurs in an equality constraint, in general a representation of its graph $\mathrm{grph}(f) = \{(z,x) | z = f(x)\}$ is needed. As shown in [53], representability of grph(f) implies representability of epi(f), but the converse implication often fails. Thus, equality constraints place more demands on representability than do function occurrences in the criterion of a program.

For example, typically this fixed charge function is used in a (cost) minimizing criterion:

$$f(x) = \begin{cases} 0, & x = 0 \\ c, & 0 < x \leq M \end{cases} \tag{2.1}$$

where $M > 0$ is a constant and $c \geqslant 0$ a fixed charge. The representation of epi(f) in (2.1) can be achieved by standard means (e.g., [35]), and in fact is the disjunctive representation of epi(f) (see [54, Section 3]). However, this function (2.1) cannot be used in an equality constraint, as a consequence of the following result.

Theorem 2.1: [53]

Assume that epi(f) is b-MIP.r and that f has a bounded domain. Then grph(f) is b-M.I.P.r iff f is continuous on its domain.

For instance, for an interperiod cash-balance constraint in which a fixed-charge may have occurred, we have $I' = I - w - f(x)$ where I is the initial cash available, I' is the cash available next period, and w is all expenses this period except for the fixed charge. This is an equality constraint. In general, it cannot be represented due to Theorem 2.1, since f is not continuous (see also examples in [53]).

Of course, the non-representability of this standard version of a fixed charge is not a practical barrier in most cases, for typically there is a minimum level $\delta > 0$ on the activity x, and values of x in $0 < x < \delta$ are not permitted. The following function g can be used to model a fixed charge in equality constraints, since g is continuous:

$$g(x) = \begin{cases} 0, & x = 0; \\ c, & \delta \leqslant x \leqslant M \end{cases} \tag{2.2}$$

In this context we again see the following subtlety of integer modelling: while there usually is a way of casting a real-world piecewise-linear situation to make it b-MIP representable, one requires a technical analysis and some sophistication in order to determine the exact features of the situation which are needed.

We studied graph representability for piecewise-linear functions on polyhedral domains in [53]. In contrast to Theorem 2.1, in which a mild and usually "fixable" condition (i.e. continuity) allows graph representability to follow from epigraph representability for functions on bounded domains, the following result holds on general (i.e. possibly unbounded) domains.

Theorem 2.2: [53]

A function f has grph(f) b-MIP representable if and only if the domain dom(f) of f is b-MIP representable, and there is a subspace L of the affine span of the domain of f, a vector $w \in L$, polytopes $Q_1,\ldots,Q_t$ in $L^\perp$, and a continuous function g with either its epigraph or hypograph b-MIP representable in $Q_1,\ldots,Q_t$ such that:

for all $x \in \text{dom}(f)$, upon putting $x = u + v$
with $u \in L$ and $v \in L^\perp$, we have $v \in Q_1 \cup \ldots \cup Q_t$ and

$$f(x) = g(v) + wu \tag{2.3}$$

The "necessary" part of Theorem 2.2 is rather restrictive, as noted in [53]. Thus, outside of bounded domains, graph representability usually fails. This result tends to recommend the explicit use of bounds, whenever they can be obtained in this setting.

3. A Concept of Structure for Mixed Integer Programs (Lecture 3)

Once one has exactly characterized the representable sets, and provided a construction which creates sharp representations, the next issue to be addressed is that of the size of the representation. Quite frequently for the common representations, there are substantial algebraic simplifications in the disjunctive representations, which allow them to become compact (see Lectures 1 and 2 from [61]). However, such simplifications do not always occur, and we are in need of systematic results to replace the ad hoc simplifications which have been observed.

Our study of compact representations proceeded with two different emphases: (1) Compact sharp representations for specially structured constraint sets, which we discuss in Sections 4 and 5 below; (2) Compact representations for general constraint sets, which are not typically sharp, and which we discuss in this section.

The study of general constraints necessarily focuses on the manner in which constraints are combined via algebraic operations, an issue we termed 'modelling linkage' earlier [63]. The operations which we studied in [55] and [60] were union, intersection, affine linear transformation (including sum and projection), and Cartesian product, as well as some more complex operations (in [55]). Each such operation is assigned a construction which parallels its action on representations of given sets. These operations are iteratively applied via composition, resulting in composite operations which are represented by composite constructions. This is the intuitive idea; however, due to technical issues (such as the restriction on recession cones for the union operation), there are limitations to how closely a composite construction can parallel a composite operation. Indeed many set operations on b-MIP.r sets lead to non representable sets, so optimally the corresponding construction yields the representable hull [54].

Since a considerable formalism is needed to carry through the above idea in a precise manner, concrete examples are needed to fix the ideas. In this respect, our discussion on pages 61-65 of Lecture 3 in [61] can be quite helpful. It will also show how composition to depth four naturally arises in a not-too-complex situation, and we will utilize it below.

In developing the theory of composite constructions, we found it useful to provide two key results which we use frequently in proofs. We cite these results next.

Let $Op(S_1,\ldots,S_t)$ be a composite set operation of t set variables $S_1,\ldots,S_t$ and let $\underline{Op}(\underline{S}_1,\ldots,\underline{S}_t)$ be the composite construction developed in [55], [60] to parallel the operation. The construction $\underline{Op}$ has, as its arguments, representations $\underline{S}_i$ of the sets S_i and it yields representations of $Op(S_1,\ldots,S_t)$ on its <u>domain</u> (i.e., where it can parallel the operation). Whenever we write $\underline{Op}(\underline{S}_1,\ldots,\underline{S}_t)$ here and below, we assumed that $(\underline{S}_1,\ldots,\underline{S}_t)$ is in the domain of $\underline{Op}$ (for conditions when this holds see (55] and [60]).

We say that $\underline{Op}$ is a <u>sharp</u> composite construction if, whenever $\underline{S}_i$ is a sharp representation of S_i for $1 \leq i \leq t$, $\underline{Op}(\underline{S}_1,\ldots,\underline{S}_t)$ is a sharp representation of the set $Op(S_1,\ldots,S_t)$.

<u>Theorem 3.1</u>: [55], [60]

$\underline{Op}$ is sharp if Op does not involve the intersection operation.

From Theorem 3.1, the intersection operation alone is responsible for loss of sharpness. When it does not occur (e.g., when only unions, sums, and projections are used) we can model the "pieces" $S_1,\ldots,S_t$ of a constraint set sharply, then combine these representations via the composite construction, and obtain a sharp representation of the entire constraint set. For example, in machine loading for t jobs on m machines where each S_i is the union of the m ways of loading job i, we can easily obtain a compact sharp representation of S_i (the usual representation is sharp). The overall loading is described by $S_1 + S_2 + \ldots + S_t$, so the composite construction yields a sharp (and compact) representation of it.

A technical notion implied by sharpness, is called (relaxation) commutativity. We say that Op is (relaxation) commutative if

$$\mathrm{Rel}[\underline{\mathrm{Op}}(\underline{S}_1,\ldots,\underline{S}_t)] = \mathrm{conv}\{\mathrm{Op}(\mathrm{Rel}(\underline{S}_1),\ldots,\mathrm{Rel}(\underline{S}_t))\} \tag{3.1}$$

(In (3.1) the $\underline{S}_i$ need not be sharp.) Here is our basic result on commutativity.

Theorem 3.2: [55], [60]

Op is (relaxation) commutative if Op does not involve both union and intersection.

(3.1) is a useful formula in later proofs. Illustrations of its use in computing relaxations are given in [55], [60], and [61, Lecture 3, pp. 68-69].

Once the technical machinery for handling composite constructions is in place, it is possible to move on to an important issue in the use of these constructions. Specifically, the same constraint set can have alternative composite constructions which all describe it. We seek to determine the trade-off between the <u>size</u> of the resulting constraint set and the <u>manner</u> in which it is <u>described</u> (i.e., the <u>syntax</u> used -- it is for this reason that we needed to have a clear distinction between the semantics and the syntax, and so built this formalism).

Distributive laws are a primary means of obtaining equivalent constraint sets via a changed representation, according to our next results.

<u>Proposition 3.1</u>: [60]

If Op(U,W) has one occurrence of the set U, and W is a vector of sets, then

$$Op(\cup_i S_i, W) = \cup_i Op(S_i, W) \qquad (3.2)$$

<u>Proposition 3.2</u>: [60]

If Op(U,W) has one occurrence of the set U, and W is a vector of sets, then

$$Op(\cap_i S_i, W) \subseteq \cap_i Op(S_i, W) \qquad (3.3)$$

with equality (=) in (3.2) if every affine linear transformation in Op is one-to-one.

N.B. Since the common transformations of sum and projection are not one-to-one, the main use of Proposition 3.2 is when transformations do not occur in Op (i.e., only unions, intersections, and Cartesian products).

Our results on reformulations via distributive laws are given next.

Theorem 3.3: [60]

For a composite construction $\underline{Op}(\underline{U},\underline{W})$ with one occurrence of the representation $\underline{U}$, and $\underline{W}$ a vector of representations:

$$\mathrm{Rel}[\underline{Op}(\vee_i \underline{S}_i,\underline{W})] \supseteq \mathrm{Rel}[\vee_i \underline{Op}(\underline{S}_i,\underline{W})] \tag{3.4}$$

In (3.4), equality (=) occurs if $\underline{Op}$ has no occurrence of intersection.

According to (3.4), one achieves a generally tighter linear relaxation by distributing with the union operation (which is paralleled by the $\vee$ construction) across the intersection operation. However, this is at the cost of a generally larger representation (i.e., it is larger except if there occur ad hoc simplifications). If there is no intersection operation one should not distribute, since the representation is already sharp (Theorem 3.1).

Theorem 3.4: [60]

For a composite construction $\underline{Op}(\underline{U},\underline{W})$ with one occurrence of the representation $\underline{U}$, and $\underline{W}$ a vector of representations:

$$\mathrm{Rel}[\underline{Op}(\wedge_i \underline{S}_i,\underline{W})] \subseteq \mathrm{Rel}[\wedge_i \underline{Op}(\underline{S}_i,\underline{W})] \tag{3.5}$$

According to (3.5), one achieves a generally tighter relaxation by undoing distribution of intersection (which is paralleled by the $\wedge$ construction) across the other operations. This actually results in a generall smaller representation, in addition, so it is always done (there is no "trade offs" to be made).

The formulas (3.4) and (3.5) are useful in bringing out the lattice nature of representations, as in Figure 15 (p. 74) of [61], and in seeing the inclusion relations among various formulations of the motivational example of [61, Lecture 3] (see pp. 63-64). We also have used these formulas to compare two different formulations of a generalization of the algorithm DP of Davis and Putnam [28] to mixed-integer programming, where DP becomes a preprocessing routine [60]. In fact, in [60, Section 6] we show that one of the two algorithms leads to representations with a superior linear relaxation, and in most cases it also is smaller, and so results in less computation and storage space. The result is overviewed in [61, Lecture 9].

In the next two sections, we turn to results for specialized constraints, where compact and sharp representations can sometimes be obtained directly, without recourse to composite constructions. This direction of research is in its early stages. It is not so much an alternate approach as it is a supplementary approach, since these sharp representations are typically only part of a larger constraint set. Due to the modular nature of composite constructions, such representations can be inserted as arguments of a construction, whether or not they arise using disjunctive techniques or any specific technique of formulation.

4. A Simplification for the Disjunctive Construction in Certain Cases

In the case of a finite union of polyhedra $P_h = \{x|Ax \geq b^{(h)}\}$ $(h=1,\ldots,t)$ with the same constraint matrix A, the unparametricized disjunctive construction gives this sharp representation for $S = P_1 \cup \ldots \cup P_t$:

$$Ax^{(h)} \geq \lambda_h b^{(h)}, \ \lambda_h \in \{0,1\} \text{ for } h=1,\ldots,t \tag{4.1}$$

$$\sum_h \lambda_h = 1$$

$$x = \sum_h x^{(h)}$$

(Here $\text{rec}(P_h) = \{x | Ax \geq 0\}$ is independent of h.) Here t linear systems are involved, as well as auxiliary variables $x^{(h)}$ (and λ_h).

Another representation of S is given by:

$$Ax \geq \sum_h \lambda_h b^{(h)} \tag{4.2}$$

$$\sum_h \lambda_h = 1, \ \lambda_h \in \{0,1\} \text{ for } h=1,\ldots,t$$

Now (4.2) is a much smaller linear system, but, as shown in [59, Section 2] it usually is <u>not</u> sharp. Our main result in [59] is a sufficient condition under which (4.2) <u>is</u> sharp.

<u>Theorem 4.1</u>: [59]

Suppose that A is of full row rank n, $x = (x_1,\ldots,x_n)$.

Then (4.2) is a sharp representation of $S = P_1 \cup \ldots \cup P_t$ if the following holds ($A = [a_i]$(rows), $b_B^{(h)} = (b_i^{(h)})$ is the subvector of $b^{(h)}$ which corresponds to the rows of B in A):

For every square nonsingular submatrix B of A, every i and every h, if $a_i\, B^{-1}\, b_B^{(h)} < b_i^{(h)}$ then there exists j such that $a_j\, b^{-1}\, b_B^{(k)} < b_j^{(k)}$ for all $k \in \{1,\ldots,t\}$. (4.3)

Admittedly, Theorem 4.1 is a highly technical result. However, by verifying [59] the sufficient condition (4.3) for unions of multidimensional intervals, unions of simplices, translations of polyhedra, and other settings we have verified the sharpness of (4.2) for these cases.

5. Specialized Constraints Arising in Production and Operations Management (Lecture 4)

In Section 3, we mentioned the use of formulations for the shop loading problem. In the last three years or so, there has been increasing interest in formulations for variations of the lot-sizing problem which occurs in production management [5], [34], [110]. This problem can be viewed as a special case of networks with fixed-charges [68], [88], [109].

This research is very useful in two ways. First, the problem studied is of interest and applicability. Second, and more importantly, new principles are being discovered which are not immediate from previous disjunctive formulations.

In [61, Lecture 4, pp. 83-84] we provided a new construction principle which subsumes, both the union construction of the disjunctive methods, and the polynomial-sized reformulation of lot sizing obtained by Eppen and Martin [34] (the latter is based on Martin's variable redefinition technique [72]). We recently have further generalized this construction and will be presenting our results at the Southampton meeting in April.

6. Spatial Embeddings (Lecture 9)

We developed an axiomatization for the concept of "embedding" a model structure [57] in Euclidean space as b-MIP.r sets, in a manner which allows a uniform method for answering arbitrary propositional logic queries regarding the structure, and with the already spatial aspects of the structure embedded "as is". We now make these concepts precise.

In a model $M = (X,D,W;\ P_1,P_2,\ldots,P_s)$ we have $X \subseteq R^n$ a subset of some Euclidean space R^n, D a set, $W \subseteq X \times D$ a non-empty subset of the Cartesian product of X and D, and each $P_i \subseteq W$. We call X the "spatial" coordinates of M, and D the "logic" or "database" coordinates. We define $7P_j = W \backslash P_j$ as the set difference of P_j from W.

The definition of an embedding has several parts. It involves non-empty b-MIP.r sets $\mathrm{IMB}(W)$, $\mathrm{IMB}(P_j) \subseteq R^t$ for some t. We require that $\mathrm{IMB}(7P_j) \overset{\mathrm{def}}{=} \mathrm{IMB}(W) \backslash \mathrm{IMB}(P_j)$ be b-MIP.r and have $\mathrm{rec}(\mathrm{IMB}(7P_j)) = \mathrm{rec}(\mathrm{IMB}(L))$ if $7P_j \neq \emptyset$. (This is a seemingly strong requirement, as typically the set-theoretic difference of b-MIP.r sets is not b-MIP.r.) We also require that $\mathrm{IMB}(P_j) = \mathrm{IMB}(W)$ for all j.

We extend the embedding to general propositional forms L in the natural way (i.e. $IMB(L_1 \vee L_2) = IMB(L_1) \cup IMB(L_2)$, $IMB(L_1 \wedge L_2) = IMB(L_1) \cap IMB(L_2)$, $IMB(L_1 \supset L_2) = IMB(\neg L_1 \vee L_2)$). Moreover we require that $IMB(\neg L) \overset{def}{=} IMB(W) \setminus IMB(L)$ be b-MIP.r with $rec(IMB(\neg L)) = rec(IMB(W))$ if $\neg L \neq \emptyset$.

The "as is" nature of the embedding for the spatial part X is expressed as follows. We require that, for all vectors $c \in R^n$ and all propositional forms L, we have:

$$\inf\{cx \mid (x,d) \in L\} = \min\{cx \mid (x,y) \in IMB(L)\}$$

Models are generally embeddable, so long as their spatial parts are b-MIP.r with "suitable" recession conditions. This is the content of our next result.

Define an <u>elementary conjunct</u> to be a conjunction $L' = \bigwedge_{j=1}^{s} P_j^{G(j)}$ of predicates P_j and their negations (i.e., $G(j) \in \{-1,1\}$ for all j, where $Pj^{-1} = \neg P_j$).

<u>Theorem 6.1</u>: [57]

Let $M = (X,D,W; P_1,\ldots,P_s)$ be a model with $X \subseteq R^n$.

If $n = 0$ (i.e., no spatial part), M is embeddable.

If $n > 1$, M is embeddable iff for every non-empty elementary conjunct L', clconv(T(L') is a polyhedron and, whenever $T(L') \neq \emptyset$, rec(clconv(T(L')) is independent of L'. Here we define:

$$T(L') = \{x \mid \text{for some } d \in D,\ (x,d) \in L'\}. \tag{6.2}$$

As we showed in [57], the concept of an embedding is broader than that of a representation, since all representable sets can be viewed as deriving from an embedding, although some b-MIP.r embeddable sets are not b-MIP.r.

When a model is embeddable, queries regarding it can be transformed in a uniform, natural way to queries regarding representable sets in Euclidean space. This potentially allows the use of Operations Research techniques to answer queries, although in the case of pure logic (i.e. $n = 0$, when there is no spatial part) algorithms from Operations Research generally specialize to become variants of known list processing routines. The actual power of Operations Research algorithms emerges when the spatial part is present in a nontrivial way, such as in capacity constraints or flow balances. Moreover, for practical implementation regarding only one query or a few queries, much of the embedding representation is stripped away and simplified.

An embedding is <u>sharp</u> if all queries are already answered by the linear relaxation. More precisely, an imbedding is sharp if there are representations $\underline{P}_j$ and $\underline{7P}_j$ for IMB(P_j) and IMB($7P_j$), which extend to propositional L via the canonical constructions of [60], and for which we have:

$$\inf\{cx \mid (x,d) \in L\} = \min\{cx \mid (x,y) \in \mathrm{Rel}(\underline{L})\} \tag{6.3}$$

for all $c \in R^n$ and all L.

<u>Theorem 6.2</u>: [57]

If a model is embeddable, it has a sharp embedding.

Unfortunately, sharp embeddings generally require a very high dimensional space, and so they are not often directly useful [57]. However means are being explored for dynamic, sequential realization of sharp embeddings (see e.g. below and in Section 8).

In many situations, the query-answering capabilities of embeddings can be extended from propositional to predicate logic [57] (see [57, Theorem 5.1] or [61, Lecture 9] for sufficient hypotheses for this extension). When this is done, Benders partitioning [10] can be viewed as a dynamic procedure for sequentially obtaining an embedding for a projected set (i.e. a set obtained by use of the existential quantifier), and it can be generalized. Moreover, a "dual" procedure to Benders' arises in connection with universal quantifiers. These procedures are discussed in [61, Lecture 9].

7. Propositional Logic and Integer Programming (Lecture 5)

In [12] we began our research into connections between logic and mixed-integer programming, by focusing on the propositioinal logic and its standard representation via linear inequalities in binary variables. We next cite a result which gives one instance of such connections.

It is natural to represent e.g. the proposition $\neg P_1 \vee P_7 \vee P_{10}$ by introducing binary variables $z(P_1)$, $z(P_7)$, $z(P_{10}) \in \{0,1\}$ and then writing:

$$(1-z(P_1)) + z(P_7) + z(P_{10}) \geq 1 \tag{7.1}$$

This is what we term the "standard representation" of propositional logic.

As it turns out, the linear relaxation of the standard representation of a set of clauses (i.e., a conjunctive normal form [96]) can be characterized, to a large extent, in terms of a known list processing algorithm called unit resolution [70]. Unit resolution also occurs as a subroutine in the algorithm of Davis and Putnam [28]; we called this subroutine "clausal chaining" or CC.

Proposition 7.1: [12]

Propositional letters set true respectively false by CC have their corresponding binary variables made identically 1 respectively identically 0 in the linear relaxation of the standard representation, and conversely, provided only that the problem is not proven inconsistent by CC. Moreover, a problem is proven inconsistent by CC exactly if its linear relaxation is empty.

Other similar interconnections can be developed (e.g. in [12, Proposition 3.2] we show that Horn clause consistency is determined by the linear relaxation), and using these one can prove further results. For instance, we showed that the linear relaxation of the second form of the Davis-Putnam algorithm was tighter than the original form of the algorithm. We then utilized this fact, together with size estimates, to show that the second form is to be preferred in most cases (see [12, Proposition 3.3] and the discussion preceeding it). These interconnections can be extended to a comparison of the Davis-Putnam algorithm to branch-and-bound [12].

It is often inconvenient (and can require exponential space) to convert a general proposition to one in conjunctive normal form (c.n.f.), in order to use algorithms like those in [28], which require c.n.f. Tseitin [101] has provided a linear time conversion of a general proposition to an equivalent (as regards satisfiability) c.n.f. in additional letters.

We addressed the issue of the relative power of CC (and hence of linear programming) on Tseitin's equivalent, as opposed to CC applied to the usual equivalent obtained by distributive laws of propositional logic. We obtained a result [12, Theorem 2.1] which gave a sufficient condition for the two linear relaxations to be of the same strength.

The analysis in [12] also illustrates several subleties. For example, Proposition 2.1 does <u>not</u> state, as might first appear, that CC and linear programming are equivalent. Indeed, linear programming can go beyond CC by locating an <u>incumbent</u> (i.e. a zero-one solution) in a consistent problem, even though not all variables are identically zero or identically one. If only they can be zero-one, an incumbent may be found by linear programming. In the latter case, CC would not be able to determine consistency, although linear programming can.

Our computer experiments reported in [12] indicate that for randomly-generated problems with over a hundred letters and clauses, branch-and bound is frequently successful in real time in determining satisfiability. In our experiments, we used only the general MIP code APEX IV.

In addition, small examples in [12] give insight as to how CC by itself is already more effective than both forward chaining (FC) and backward chaining (BC), even on Horn clauses. Further examples were given in [61, Lecture 5]. Since CC can be implemented in linear time by adapting the algorithm of Dowling and Gallier [33], this fact indicates that CC is preferable to both FC and BC for propositional logic. Moreover, as CC is only one subroutine of the Davis-Putnam algorithm, the relative weakness of the popular methods FC and BC (often used in expert systems [42], [43]) becomes evident.

8. New Techniques for Predicate Logic (Lecture 8)

Our approach to predicate logic is in its early stages of development. The approach is begun in [58], continued in [61, Lecture 8], and is currently being extended, prior to computer testing. We are seeking to provide more efficient implementations of logic programming, which can also treat some instances of non Horn clauses, and the kinds of linear constraints which are so well handled by Operations Research methods (e.g. budget and factor constraints, and flow balances).

We will summarize here the nature of our approach, rather than cite technical results.

Predicate logic queries on a domain can be reduced to propositional logic, thus allowing the use of efficient routines for propositional logic. However, the typical reductions as e.g. [58, Theorem 1.1] require exponential space even for decidable fragments of predicate logic.

Instead of making the complete reduction at once, one can instead proceed through a sequence of increasingly accurate "approximations" to the propositional logic equivalent. What is needed in this context are: (1) Fathoming tests, which may be able to pick up crucial information about the original query from a given approximation; and (2) A method of improving the accuracy of the approximation, when the fathoming tests fail to extract information.

We implemented this program in [58]. There we obtained two fathoming tests, one for detecting inconsistency and one for detecing consistency. We also introduced the concept of "blocked" and "unblocked" truth valuations, to pinpoint exactly where increased accuracy is needed. A flow chart for the algorithm framework is given in [61, Lecture 8].

The specific representation studied was not successful, in part due to our use of linear programming to implement it.

For handling logic alone, linear programming is at a relative disadvantage, due to its need to carry along and update irrelevant structures, such as bases and other arrays. List processing routines are more direct and faster. However, by studying what linear programming achieves on a pure logic problem we have often found it possible to parallel this action by a more efficient list processor, in the spirit in which graph theory is used to make linear programming more efficient in networks.

We are currently continuing our work on alternate representations of logic, and combining that with a study of simplifications in linear programming for its application to the specific new representations. Our goal is to obtain even more efficient list processors for pure logic, and subsequently, to bring in additional nonlogical constraints which will require more direct use of programming techniques.

10. Iterative Definitions Via Mixed Integer Programming

In [56], we explored the following kind of iterative process, for defining a limit vector $\bar{x} = (\bar{x}_1,\ldots,\bar{x}_n) \in R^n$.

An initial vector $x^{(0)} \in R^n$ is given, together with b-MIP.r functions $f_k(x)$ and an index set K_i for determining values of the i-th coordinate x_i of x. Given $x^{(t)}$, we compute the i-th coordinate of the next update $x^{(t+1)}$ by:

$$x_i^{(t+1)} = \max\{f_k(x^{(t)}) \mid k \in K_i)\} \qquad (10.1)$$

for i=1,...,h. We desire to find $\bar{x}$ where

$$\bar{x}_i = \lim_t x_i^{(t)} \tag{10.2}$$

when the limit vector $\bar{x}$ exists.

We assume that all functions f_h are <u>monotone</u> in the sense that, whenever $x \geqslant x'$ (coordinatewise), also $f_k(x) \geqslant f_k(x')$.

As we shall see, under mild hypotheses the desired limit can be computed via this mixed-integer program:

$$\min \sum_{i=1}^{n} a_i x_i \tag{10.3}$$

$$\text{subject to} \quad \left.\begin{array}{ll} x_i \geqslant f_k(x) & \text{for } k \in K_i \\ & x_i \geqslant x_i^{(0)} \end{array}\right\} \quad i=1,\ldots,n$$

<u>Theorem 10.1</u>: [56]

If all the functions f_k are b-MIP.r and monotone, all $a_i > 0$ in (10.3), and $\bar{x}$ exists and $\bar{x} \geqslant x^{(0)}$, then a unique optimal solution x^* to (10.3) exists and $\bar{x} = x^*$.

The content of Theorem 10.1 is that the potentially infinite process of iteratively computing $x^{(0)}, x^{(1)}, x^{(3)},\ldots$ via (10.1), and then "computing" via (10.2), can be replaced by solving one MIP (10.3). (An example in [56] shows that the process can be infinite.)

The result of [56] was motivated by the computation of "certainty factors" for expert systems, and the use of "inexact reasoning" in these systems (see the discussion in [56]).

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71. Lowe, James K., Modelling With Integer Variables, Ph.D. Thesis, Georgia Institute of Technology, March 1984.

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74. Martin, R. Kipp, "A Sharp Polynomial Size Linear Programming Formulation of the Minimum Spanning Tree Problem," Graduate School of Business, University of Chicago.

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110. Zangwill, W. I., "A Backlogging Model and a Multi-Echelon Model of a Dynamic Economic Lot Size Production System -- A Network Approach," Management Science 15 (1969) 506-527.

Publication Activity

July 1983 to December 1986

Papers Published

1. "Cluster Sets of Vector Series," with D. F. Karney, Advances in Applied Mathematics 5 (1984) 470-475.

2. "Extensions of a Theorem of Balas," with C. E. Blair, Discrete Applied Mathematics 9 (1984) 11-26.

3. "Modeling with Integer Variables," with J. K. Lowe, Mathematical Programming Studies 22 (1984) 167-184.

4. "Experimental Results on the New Techniques for Integer Programming Formulation," with J. K. Lowe, Journal of the Operational Research Society 36 (1985) 393-403.

5. "The Polynomial Hierarchy and a Simple Model for Competitive Analysis," Mathematical Programming 32 (1985) 146-164.

6. "Computational Complexity of Some Problems in Parametric Discrete Programming," with C. E. Blair, Mathematics of Operations Research 11 (1986) 241-260.

7. "Constructive Characterizations of the Value Function of a Mixed Integer Program: I," with C. E. Blair, Discrete Applied Mathematics 9 (1984) 217-233.

8. "Constructive Characterizations of the Value Function of a Mixed Integer Program: II," with C. E. Blair, Discrete Applied Mathematics 10 (1985) 277-240.

9. "Some Results and Experiments in Programming Techniques for Propositional Logic," with C. E. Blair and J. K. Lowe, Computer and Operations Research 13 (1986) 633-645.

Papers Submitted for Publication

1. "Representability in Mixed Integer Programming, I: Characterization Results," accepted to Discrete Applied Mathematics.

2. "Representability of Functions."

3. "Computation-Oriented Reductions of Classical Predicate to Propositional Logic."

4. "On Monotone Chaining Procedures of the CF Type."

5. "Alternative Formulations of Mixed Integer Programs."

6. "A Simplification for Disjunctive Formulations."

7. "Ten Lectures on Mixed Integer Programming Formulations for Logic-Based Decision Support."

8. "Solving Propositional Satisfiability Problems," with J. Wang.

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January 1987

Personal

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Marital Status:
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Education

B.S. 1964 — Columbia University
School of Engineering
Department of Industrial Engineering

1964-1966 — Cornell University
School of Engineering
Department of Industrial Engineering and Operations Research
(completed Comprehensive Examination)

Ph.D. 1969 — Cornell University
Department of Mathematics
Professor Anil Nerode, Advisor

Experience

University of Minnesota
School of Mathematics
September 1969-August 1972
Assistant Professor

Experience (continued)

Carnegie-Mellon University
Graduate School of Industrial Administration and
Department of Mathematics
Associate Professor
September 1972-August 1976
Professor
September 1976-June 1980

Georgia Institute of Technology
College of Management and
School of Industrial and Systems Engineering
Professor
September 1978 to date

Research Interests

I am interested in decision support systems which involve combinations of techniques from artificial intelligence and quantitative methods. My current research emphasizes model formulation in mixed integer programming, the structure of long-term memory, and techniques for logic-based support systems.

Teaching Interests

Research interests, plus production and applications of Operations Research techniques; management strategy.

Journals and Reviews

Member of the Editorial Board (Associate Editor), Discrete Applied Mathematics.

Referee for Operations Research, Management Science, Mathematical Programming, and Discrete Mathematics, other journals. Reviewer for the National Science Foundation.

Grants and Fellowships

Ford A Fellowship 1964-1966
NSF Graduate Fellowship 1966-1969
NSF Research Grant GP 21067, Principal Investigator, 1971-1972
(Grant Awarded 1970)
NSF Research Grant GP-37510X, Associate Investigator, 1973-1975

Grants and Fellowships (continued)

NSF Research Grant MCS76-12026, Co-Principal Investigator, 1976-1978
Research Fellowship, January - June 1977, from the Center for Operations Research and Econometrics, Belgium
NSF Research Grant ENG-79000284, Principal Investigator, 1979-1980
NSF Research Grant ECS-8001763, Principal Investigator, 1980-1982
Senior U.S. Scientist Award of the Alexander von Humboldt Foundation, January to June, 1983
NSF Research Grant MCS-804075, Principal Investigator, 1983-86
NSF Research Grant DMS-8513970, Principal Investigator, 1986-88

Organizational Responsibilities

Representative to the Faculty Senate from the business school, 1977-1978
Organizer of the Operations Research Seminar for the business school, 1977-1978
Representative of the management college to the Seminar on Operations Research, 1978-1983 and 1983-1984 (co-spondored with the school of Industrial and Systems Engineering and the School of Mathematics)
Member of the Program Committee of the symposium in honor of R. J. Duffin Constructive Approaches to Mathematical Models, July 10-15, 1978
Co-or anizer (with Cedric Suzman) of the Colloquia on Strategic Planning, September 28, 1979, and October 10, 1980
Imple enter and morning moderator, Third Colloquium on Strategic Planning, October 1981
Member of the Organizing Committee of the 1981 International Symposium on Semi-infinite Programming and Applications
Member of the International Program Committee for the XI International Symposium on Mathematical Programming, Bonn, August 23-27, 1982
Member of the Teaching Evaluation Committee in the College of Management, 1980-1981
Chairman and member of the Personnel Committee in the College of Management, 1980-1982, 1983-1985
Secretary of the Personnel Committee in the College of Management, 1983-84
Chairman and member of the Ph.D. Committee in the College of Management, 1983-84
Member of the Institute Self-Study Committee on Special Activities, 1982-84
Member of the International Program Committee for the XII international Symposium on Mathematical Programming, August 1985
Plenaries and tutorials for ORSA/TIMS Atlanta, November 1985
Track chairperson for "OR/AI Interface," ORSA/TIMS Miami Beach,
Management Science Co-ordinator, College of Management, Georgia Institute of Technology
College of Management Representative to Institute Committee on Computer Integrated Manufacturing Systems

Hobbies and Personal Interests

Swimming; weight training; poetry and theater.

Published Articles by Subject Area

Logic and the Foundations of Mathematics

"Consistency Statements in Formal Theories", Fundamentae Mathematicae, LXXXII (1971), pp. 17-40.

"Non-effectiveness in S. Orey's Arithmetical Compactness Theorem", Zeithschrift f. math. Logik and Grundlagen d. Math., Bd. 17, 1971, pp. 285-289.

"Redundancies in the Hilbert-Bernays Derivability Conditions for Godel's Second Incompleteness Theorem," Journal of Symbolic Logic 38 (1973), pp. 359-367.

"Experimental Logics and Δ_2^0-Theories", Journal of Philosophical Logic 4 (1975) 253-267.

Convex and Semi-Infinite Programming

"On Semi-infinite Systems of Linear Inequalities", with K. O. Kortanek, Israel Journal of Mathematics 10 (1971) 252-258.

"Lagrange Dual Problems with Linear Constraints on the Multipliers", with C. E. Blair, Constructive Approaches to Mathematical Models, C. V. Coffman and G. Fix (eds.), Academic Press, 1979, 137-152.

"Lagrangean Functions and Affine Minorants", with R. J. Duffin, Mathematical Programming, Study 14 (1981) 48-60.

"A Limiting Lagrangean for Infinitely-constrained Convex Optimization in R^n", Journal of Optimization Theory and Applications, 33 (1981) 479-495.

"A Limiting Infisup Theorem," with C. E. Blair and R. J. Duffin, Journal of Optimization Theory and Applications 37 (1982) 163-175.

"Duality in Semi-infinite Programming," with R. J. Duffin and L. A. Karlovitz, Semi-Infinite Programming and Optimization, edited by A. V. Fiacco and K. O. Kortanek, Springer-Verlag, 1983.

"Uniform Duality in Semi-Infinite Convex Optimization," Mathematical Programming 27 (1983) 144-145.

Mathematical Programming (General)

"A Note on Some Classical Methods in Constrained Optimization and Positively Bounded Jacobians", with K. O. Kortanek, Operations Research 15 (1967) 964-969.

"Linear Programs Dependent on a Single Parameter," Discrete Mathematics 6 (1973) 119-140.

Published Articles (continued)

"An Exposition on the Constructive Decomposition of the Group of Gomory Cuts and Gomory's Round-Off Algorithm", with K. O. Kortanek, Cahiers du Centre d'Etudes de Recherche Operationnelle 2 (1971) 63-84.

"Asymptotic Linear Programming", Operations Research 21 (1973) 1128-1141.

"On Algorithms for Discrete Problems", Discrete Mathemtics 7 (1974) 273-280.

"Experimental Results on Hillier's Linear Search", with T. H. C. Smith, Mathematical Programming 9 (1975) 371-376.

"Some Basis Theorems for Integral Monoids", Mathematics of Operations Research 3 (1978) 145-154.

"Some Relaxation Methods for Linear Inequalities", Cahiers du Centre d'Etudes de Recherche Operationnelle 21 (1979) 43-53.

"An Exact Penalty Method for Mixed Integer Programs", with C. E. Blair, Mathematics of Operations Research 6 (1981) 14-18.

"Some Influences of Generalized and Ordinary Convexity in Disjunctive and Integer Programming," Generalized Concavity in Optimization and Economics, Academic Press, 1981, pp. 689-699 (ISBN-0-12-621120-5).

"Cluster Sets of Vector Series," with D. F. Karney, Advances in Applied Mathematics 5 (1984) 470-475.

"Some Results and Experiments in Programming Techniques for Propositional Logic," with C. E. Blair and J. K. Lowe, Computers and Operations Research 13 (1986) 633-645.

Cutting-Plane Theory and Problem Formulation

"Comments on Integer Hulls of Two Linear Constraints", Operations Research 19 (1971) 1061-1069.

"On An Algorithm of Gomory", with K. O. Kortanek, SIAM Journal on Applied Mathematics 21 (1971) 55-59.

"Canonical Cuts on the Unit Hypercube", with E. Balas, SIAM Journal on Applied Mathematics 23 (1972) 61-69.

"On Defining Sets of Vertices of the Hypercube by Linear Inequalities", Discrete Mathematics 11 (1975) 119-124.

Published Articles (continued)

"A Generalization of a Theorem of Chvatal and Gomory", pp. 313-332 in Nonlinear Programming 2, edited by O. L. Mangasarian, R. R. Meyer, and S. M. Robinson, Academic Press, New York, 1975.

"Cutting-planes Theory: Disjunctive Methods", Annals of Discrete Mathematics 1 (1977) 292-330.

"Cutting-planes for Complementary Constraints", SIAM Journal on Control and Optimization 16 (1978) 56-62.

"Cutting-plane Theory: Algebraic Methods", Discrete Mathematics 23 (1978) 121-150.

"A Converse for Disjunctive Constraints", with C. E. Blair, Journal of Optimization Theory and Its Applications 25 (1978) 195-206.

"Minimal Ineqalities", Mathematical Programming 17 (1979) 1-15.

"Two Lectures on the Theory of Cutting-planes", for Combinatorial Optimization, edited by N. Christofides et al., John Wiley and Sons, Ltd.

"Representations of Unbounded Optimizations as Integer Programs", Journal on Optimization Theory and Its Applications 30 (1980) 339-351.

"An Introduction to the Theory of Cutting-planes," Annals of Discrete Mathematics 5 (1979) 71-95.

"A Cutting-plane Game for Facial Disjunctive Programs," SIAM Journal on Control and Optimization 18 (1980) 264-281.

"Strengthening Cuts for Mixed Integer Programs", with E. Balas, European Journal of Operations Research 4 (1980) 224-234.

"Extensions of a Theorem of Balas," with C. E. Blair, Discrete Applied Mathematics 9 (1984) 11-26.

"Modelling with Integer Variables, with J. Lowe, Mathematical Programming Studies 22 (1984) 167-184.

"Experimental Results on the New Techniques for Integer Programming Formulation," with J. Lowe, Journal of the Operational Research Society 36 (1985) 393-403.

Computational Complexity

"There Cannot be any Algorithm for Integer Programming with Quadratic Constraints", Operations Research 21 (1973) 221-224.

"The Simplex Algorithm with the Pivot Rule of Maximizing Criterion Improvement", Discrete Mathematics 4 (1973) 367-378.

Published Articles (continued)

"Trivial Integer Programs Unsolvable by Branch and Bound", Mathematical Programming 6 (1974) 105-109.

"Bracketing Discrete Problems by Two Problems of Linear Optimization", in Operations Research Verfahren (Methods of Operations Research) XXV, 1977, pp. 205-216, Verlag Anton Hain, Meisenheim an Glan.

"The Polynomial Heirarchy and a Simple Model for Competitive Analysis," Mathematical Programming 32 (1985) 146-164.

"Computational Complexity of Some Problems in Parametric Discrete Programming," with C. E. Blair, Mathematics of Operations Research, 11 (1986) 241-260.

Value Functions and Sensitivity Analysis

"The Value Function of a Mixed Integer Program: I", with C. E. Blair, Discrete Mathematics 19 (1977) 121-138.

"The Value Function of a Mixed-Integer Program: II", with C. E. Blair, Discrete Mathematics 25 (1979) 7-19.

"The Value Function of an Integer Program," with C. E. Blair, Mathematical Programming 23 (1982) 237-273.

"Constructive Characterizations of the Value Function of a Mixed Integer Program, I" Discrete Applied Mathematics 9 (1984) 217-233.

"Constructive Characterizations of the Value Function of a Mixed Integer Program, II" with C. E. Blair, Discrete Applied Mathematics 10 (1985) 227-240.

Book Review

M. R. Hestenes' Optimization Theory: The Finite-Dimensional Case, reviewed in Bulletin of the American Mathematical Society (83), May 1977, pp. 324-334.

Professional Newsletter

"Some Roads Hardly Taken," p. 1-3 of OPTIMA, July 1981, the newsletter of the Mathematical Programming Society.

"The Programming of (Some) Intelligence: Opportunities at the OR/AI Interface," p. 1-3 of OPTIMA, January 1985.

Submitted for Publication

"Representability in Mixed Integer Programming, I: Characterization Results," accepted to Discrete Applied Mathematics.

Submitted for Publication (continued)

"Representability of Functions," revision under consideration.

"Computation-Oriented Reductions of Classical Predicate to Propositional Logic."

"On Monotone Chaining Procedures of the CF Type."

"Joint Product Cost Allocation in the Context of Cost-Plus Pricing Determinations with Nonuniform Markups," with A. Schneider.

"Alternative Formulations of Mixed Integer Programs"

"A Simplification for Disjunctive Formulations"

"Ten Lectures on Mixed Integer Programming Formulatons for Logic-based Decision Support."

"Solving Propositional Satisfiability Problems," with J. Wang.

Other Work in Preparation

"Self-referential Structures for Intelligence"

"Automata with Behavioral Strategies."

"Approaches to Intelligent Decision Support," a volume to appear in the series Annals of Discrete Mathematics, to appear in 1987 (volume editor).

Unpublished Work

"The Line Executive as Planner: Proceedings of the Second Annual Georgia Tech Colloquim on Strategic Planning," co-edited with C. Suzman, available as a Georgia Tech report.

"Some Remarks on the Role of Applications Software in the Computer and Information Industries," April 1983.

Invited Talks Since 1980

"Sensitivity Analysis for Integer Programs," ORSA/TIMS National Meeting at Colorado Springs, November 10-12, 1980.

"Integer Analogues", Mathematical Sciences Department, University of Delaware, November 1980.

"Sensitivity Analysis for Mixed Integer Programs", CORS/ORSA/TIMS Joint National meeting, Toronto, May 3-6, 1981.

Invited Talks Since 1980 (continued)

"Some Influences of Generalized and Ordinary Convexity in Disjunctive and Integer Programs", NATO Advanced Study Institute on Generalized Concavity in Optimization and Economics, Vancouver, August 4-15, 1980.

"Computational Complexity and the Value Function", with C. E. Blair, ORSA/TIMS meeting in Houston, October 14-16, 1981.

"Necessary and Sufficient Constraint Qualifications", 1981 International Symposium on Semi-Infinite Programming and Its Applications, Austin, Texas, September 8-10, 1981.

"Duality in Semi-infinite Linear Programming", 1981 International Symposium on Semi-Infinite Programming and Its Applications, Austin, Texas, September 8-10, 1981.

"Modelling With Integer Variables," University of Iowa, April 1982.

"Modelling With Integer Variables", with J. Lowe, XI International Symposium on Mathematical Programming, Bonn, August 1982.

"Some Recent Experimental Results on Integer Modelling", Mathematics Institute at Oberwolfach, Federal Republic of Germany, January 1983.

"Theory of Value Functions for Discrete Variable Problems: An Overview", University of Bonn, Federal Republic of Germany, February 1983.

"Some Recent Experimental Results on Integer Modelling", The Technion (Israel Institute of Technology), Israel, April 1983.

"A Problem in Competitive Analysis and the Polynomial Heirarchy", University of Bonn, Federal Republic of Germany, April 1983.

"The Value Function of Pure and of Mixed Integer Programs", Stichting Mathematics Center, Amsterdam, May 1983.

"Modelling with Integer Variables", Erasmus University, Rotterdam, May 1983.

"Modelling with Integer Variables", Mathematical Programming Study Group, London, at the London School of Economics, June 1983.

"A Problem in Competitive Analysis and the Polynomial Hierarchy," GSIA, Carnegie-Mellon University, October 1983.

"Applications Software: Some Comments," University of Illinois, October 1983.

"Computational Experiments with Integer Representation," ORSA/TIMS Orlando meeting, November 7-9, 1983.

"Applications Software: Comments," Graduate School of Business, University of Texas, February 1984

Invited Talks Since 1980 (continued)

"Representability and Modelling in Mixed Integer Programming," Curriculum in Operations Research, University of North Carolina, March 1984.

"Representability of Nonlinear Sets and a Lattice of Relaxations," University of Bonn, June 1984.

"A Problem of Competitive Analysis and the Polynomial Hierarchy," TIMS XXVI International Meeting, Copenhagen, June 1984.

"Symmetric Duality," presented by co-author D. F. Karney, at the International Symposium on Infinite Dimensional Linear Programming, September 1984.

"Expert Systems and Mixed-Integer Programming," ORSA/TIMS Meeting at Dallas, November 1984.

"Integer Programming and Propositional Logic," at the Twelfth International Symposium on Mathematical Programming, M.I.T., August 1985.

"Distributivity in the Lattice of Program Formulations," ORSA/TIMS Atlanta, November 1985.

"Combinatorial Optimization Formulations for a Fragment of Predicate Logic," ORSA/TIMS Los Angeles, April 1986.

"Mixed-Integer Model Formulation for Logic-Based Decision Support," a series of ten lectures at Rutgers University, May 1986.

"An MIP Relaxation Calculus and Boolean Logic," ORSA/TIMS meeting at Miami Beach, October 1986.

"A Predicate Logic Problem Solving Method Utilizing Operations Research," University of Iowa, September 1986.

"The Concept of Structure in Mixed Integer Programming," Combinatorial Optimization '87, University of Southampton, April 1987.

"Generalizations of Benders' Partitioning," ORSA/TIMS New Orleans meeting, May 1987.